# U. P. HIGHER EDUCATION SERVICES COMMISSION, PRAYAGRAJ

**Syllabus**

## MATHEMATICS

**(Subject Code-75)**

**Unit- 1. ALGEBRA :**

Cyclic groups, permutation groups, Cayley's Theorem, Fundamental Theorem of homomorphism, Group action, class equation, Sylow theorems and their applications; Rings, ideals, prime and maximal ideals, quotient rings, unique factorization domain, principal ideal domain, Euclidean domain, Polynomial rings, Fields, finite fields, field extensions, Modules, Noetherian modules, Hilbert basis theorem.

**Unit-2. LINEAR ALGEBRA :**

Linear transformations, Algebra of linear transformations, kernel, range, Rank-Nullity Theorem, Matrix representation of a linear transformation, change of bases, linear functionals, dual spaces, rank, system of linear equations, eigen values and eigen vectors, Cayley – Hamilton Theorem, diagonalization, Hermitian, Skew- Hermitian and Unitary matrices, Finite dimensional inner product spaces, Gram Schmidt-Quadratic forms, reduction and classification of quadratic forms. Rational and Jordan canonical forms

**Unit-3. ANALYSIS :**

Sequence and series of functions, uniform convergence, Fourier series, Riemann integral, improper integrals, functions of bounded variations, Lebesgue measure, measurable functions, Lebesgue integral, Multivariable calculus- functions of several variables, directional derivative, partial derivative and total derivative, maxima and minima, Elements of metric spaces- convergence, continuity, uniform continuity, compactness, connectedness, completeness.

Normed linear spaces, Banach spaces, open mapping theorem, closed graph theorem, Hahn- Banach theorem, Hilbert spaces, Orthogonal complement of a subspace in a Hilbert space, Orthogonal basis, Gram-Schmidt orthogonalization process.

**Unit-4. COMPLEX ANALYSIS :**

Analytic functions, Cauchy - Riemann equations, Cauchy's theorem, Morera's theorem, Liouville's theoren, Cauchy's Integral formula, zeros of analytic functions, Taylor series, Laurent series, Calculus of residues, contour integration, conformal mappings, Mobius transformations.

**Unit-5. TOPOLOGY & DIFFERENTIAL GEOMETRY:**

Basic concepts of Topology, Continuity, Homeomorphism, connectedness, compactness, countability, separation axioms, subspaces, product spaces, quotient spaces, Tychonoff's theorem, Urysohn's Metrization theorem.

Space curves - Their curvature and torsion, Serret - Frenet formulae, First and Second fundamental forms, Gaussian curvatures, Principal directions and principal curvatures, Geodesics, Manifolds.

**Unit-6. DIFFERENTIAL EQUATIONS :**

Existence and uniqueness of solutions of initial value problems for first order ordinary differential equations, singular solutions of first order ordinary differential equations, General theory of homogeneous and non-homogenous linear ordinary differential equations, Strum-Liouville boundary value problem.

Lagrange and Charpit methods for solving first order partial differential equations, General solution of higher order partial differential equations with constant coefficients, classification of second order partial differential equations, Method of separation of variables for Laplace, Heat and Wave equations.

**Unit-7. MATHEMATICAL METHODS:**

Calculus of variations- Linear functionals, Necessary and sufficient conditions for extrema, Euler-Lagrange equation, Linear integral equations of Fredholm and Volterra type, solution by successive substitutions and successive approximations, solution of integral equations with separable Kernels, Laplace transforms.

**Unit-8. MECHANICS:**

Generalised coordinates, Lagrange's equations, Hamilton's canonical equations, Hamilton's variational principle, Euler's dynamical equations of motion of a rigid body, theory of small oscillations, Poisson bracket, Canonical transformations.

Equation of continuity in fluid motion, Euler's equation of motion for perfect fluid, Two- dimensional fluid motion, complex potential, source and sink, doublets, motion of sphere in perfect fluid and motion of liquid past a sphere, vorticity, Navier-Stokes' equations for viscous flows.

**Unit-9. OPERATION RESEARCH & NUMERICAL ANALYSIS :**

Linear programming problem, solution of linear programming problem by graphical and simplex methods, M-technique, Two-phase method, Dual problem and duality theorem, convex set theory, balanced and unbalanced transportation problems, Hungarian method for solving assignment problems, Game theory.

Numerical solutions of algebraic equations, Fixed point iteration and Newton-Raphson methods, Solution of systems of linear algebraic equations using Gauss elimination and Gauss-Seidel methods, Finite differences, Lagrange, Hermite and spline interpolations, Numerical differentiation and integration, Numerical solutions of ordinary differential equitation using Picard, Euler, modified Euler and Runge-Kutta methods.

**Unit-10. VEDIC MATHEMATICS AND NUMBER THEORY :**

Contributions of ancient Indian mathematicians, Basic concepts of vedic mathematics, Contribution of Ramanujan in number theory, Fundamental theorem of arithmetic, arithmetical functions, Mobious inversion, Congruences, Chinese remainder theorem, Quadratic reciprocity law and its applications.

**NOTE :** Syllabus contains 10 Units. In preparation of question paper, it is suggested that at least 06 questions should be asked from each unit.